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EXPERIMENTAL INVESTIGATIONS OF FILTRATION OF RELAXING LIQUIDS IN HETEROGENEOUS POROUS MEDIA

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Experimental investigations of filtration of relaxing liquids in heterogeneous porous media are performed. Models for describing the obtained results are proposed that can be used in practical calculations.

Recently, rheotechnology has gained wide use in the petroleum industry [1]. It employs working fluids that have non-Newtonian properties to act on workable beds, which determines the interest in the filtration of relaxing liquids in porous media.

It has been shown [2] that elastic effects in porous media increase the filtration resistances in a steady-state flow as $1 + A(\tau v/r)^2$, where v is the filtration rate, τ is the relaxation time, r is the pore average size, and A is a constant coefficient. Viscoelastic effects that occur in filtration in porous media have also been investigated [3-5]. However, in all these works, no consideration was given to the influence of the heterogeneity of a porous medium on filtration of relaxing liquids; therefore, the results of experimental investigations devoted to this problem are presented here.

The investigations were carried out on a laboratory setup that incorporated the following basic elements: a model of a bed, a PVT bomb, a hydraulic press, an ultrathermostat, and measuring instruments.

The experiments were conducted according to the following scheme.

A high-pressure column, which was a hollow steel cylinder with quartz sand glued inside, a working-portion length of 1.1 m, and an inner diameter of 0.033 m, was filled with quartz sand, depending on the aim of the experiment: with a single fraction of $(0.314-0.25)\cdot 10^{-3}$ m, with a mixture of fractions of $(0.314-0.25)\cdot 10^{-3}$ m and $(0.14-0.005)\cdot 10^{-3}$ m (a microheterogeneous porous medium), and in the form of layers with different permeabilities (a macroheterogeneous porous medium; in the first series of experiments, the layer-permeability ratio $K_0 = 5$), which was achieved using a separating partition that was gradually removed with filling of the column. The filling was carried out by vertical vibration compaction.

In the porous medium, the pore volume and the air permeability were determined by the known procedure (after binding of the entire setup). The air permeability was 0.12 μm^2 for the microheterogeneous porous medium and 3.0 μm^2 for the macroheterogeneous medium in all the experiments; to compare the results, homogeneous porous media were selected, respectively, with permeabilities of 0.12 and 3.0 μm^2. Then the porous medium was vacuum-treated with constant thermostatic control.

In the PVT bomb, whose volume was 3.4 liter, we prepared the liquid in question: a diluted polyacrylamide (PAA)-based polymer solution, a water-oil emulsion, and a gas-liquid system in the subcritical region [6]. The results of rotational viscosimetry for the PAA and the emulsion are given in Fig. 1.

The filtration rate as a function of the pressure drop was measured during filtering.

In the first series of experiments, we investigated filtration in the homogeneous and heterogeneous porous media of water, the polymer system, the water-oil emulsion, and the gas-liquid system in the subcritical region. The basic forms of the dependences obtained are shown in Fig. 2, from which it is evident that in water filtration (curve 1) in the homogeneous and heterogeneous media, it is linear in all cases (which also indicates that for the

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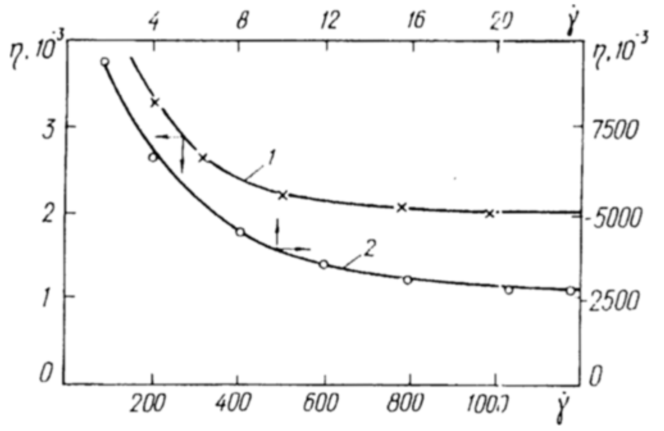

Fig. 1. Results of rotational viscosimetry: 1) PAA solution; 2) water-oil emulsion. η, Pa·sec; $\dot{\gamma}$, 1/sec.

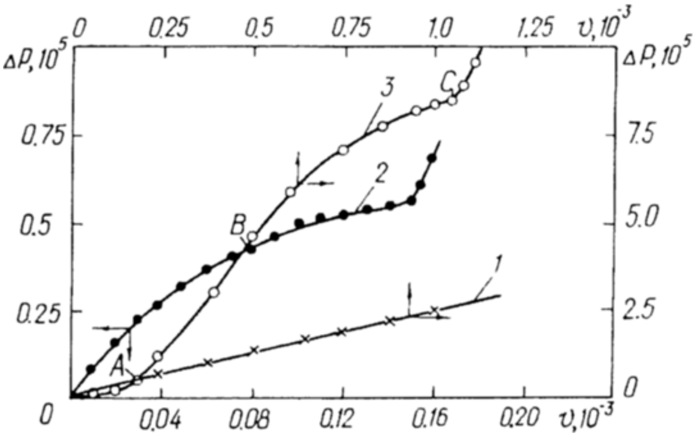

Fig. 2. ΔP (Pa) as a function of v (m/sec) for: 1) water in homogeneous and heterogeneous porous media; 2, 3) a relaxing liquid in homogeneous and heterogeneous porous media. ΔP, Pa; v, m/sec.

filtration rates investigated in the experiments, no turbulization of the flow occurs). For motion of relaxing liquids, it is of a nonlinear, nearly exponential character in the homogeneous medium (curve 2) and S-shaped in the heterogeneous porous media (line 3). It is pertinent to note that in the microheterogeneous porous medium, despite equality of the permeabilities, there are high- and low-permeability directions in each cross-section [7].

To explain the results obtained, the S-shaped curves were treated using asymptotic coordinates [8]. New coordinates were determined in the following manner: $Y = (\Delta P - \Delta P_0)/(\Delta P_\infty - \Delta P_0)$, $x = v/v_\infty$, where ΔP, ΔP_0, and ΔP_∞ are, respectively, the current, initial, and final values of the pressure drop; v and v_∞ are, respectively, the current filtration rate and the final filtration rate.

The initial curves are presented in Fig. 3a, and the results of the treatment, in Fig. 3b; they show the similarity of the three curves (in the asymptotic coordinates, they are all virtually on one curve (see Fig. 3b)), i.e., the processes are identical in all the three cases. This is due to the fact that the structural elements that form these systems, i.e., polymer balls, globules, and nuclei of the gas, have similar sizes – 10^{-6}–10^{-7} m [6, 9,5] – and elastic properties. Therefore, the results of relaxing liquid motion in the heterogeneous porous medium can be attributed to the elastic properties of these systems. When moving at a low velocity throught a system of pores of different sizes (to point A, Fig. 2, curve 3, Newtonian flow with maximum Newtonian viscosity), the liquid has time to relax (the relaxation time is smaller than the transient time); the liquid shows practically no elastic properties. Once a certain threshold velocity has been overcome (non-Newtonian flow), the structural elements of the liquid have no time to relax, changing their shape in pores of different size, and elastic forces start to prevail over viscous resistance forces, which leads to an overall increase in the resistance of the medium (to inflection point

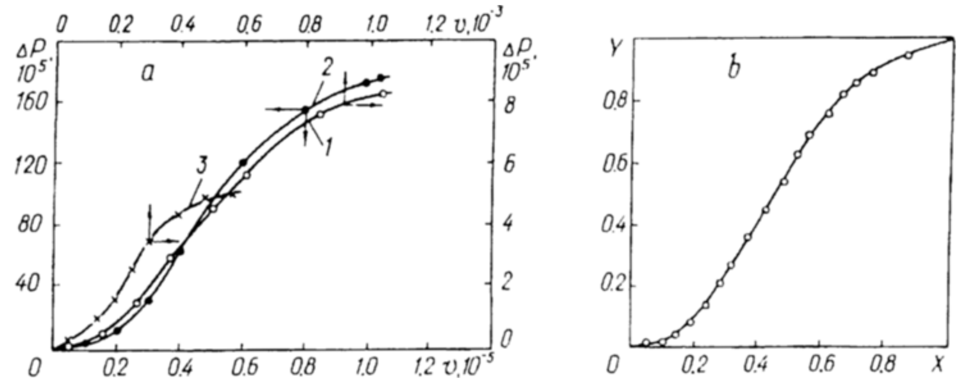

Fig. 3. ΔP as a function v (a) and Y as a function of X (b) in filtration in a heterogeneous porous medium: 1) PAA solution; 2) water-oil emulsion; 3) gas liquid system in the subcritical region.

B, Fig. 2, curve 3). Then because of the weakening of intermolecular bonds and the predominant orientation of the structural elements of the liquid along the lines of the current [10], the resistance of the medium decreases and, after a certain threshold velocity (point *C*, Fig. 2, curve 3), there is again a Newtonian flow (with the minimum Newtonian viscosity) and the $\Delta P - v$ dependence is linear.

As has been noted above, for filtration of viscoelastic liquids, the resistance grows as $1 + A(\tau v/r)^2$ and the law of filtration correspondingly appears as [5, 11]:

$$\frac{\partial P}{\partial x} = -\frac{\eta_0}{k} v\,(1 + AKch)\,,$$

where $\partial P/\partial x$ is the pressure gradient; $Kch = \tau v/r$ is a parameter that characterizes the resistance of the system.

Clearly the $\Delta P - v$ dependences obtained by us are not described by this law.

Thus, the filtration of relaxing liquids in a homogeneous porous medium (Fig. 2, curve 2) can be described by a power law [12]; in this case, the curve obtained is approximated by the model:

$$v = 0.95\Delta P^{1.53}\,.$$

As for a quantitative description of the *S*-shaped curves, the law of filtration can be written in Darcy's form but with a variable viscosity, dependent on the parameter *Kch*:

$$v = -\frac{k}{\eta\,(Kch)}\,\frac{\partial P}{\partial x}\,.$$

By expanding the function $\eta(Kch)$ into a series we obtain

$$\eta = \eta_0\,(a + bKch + cKch^2)\,, \qquad (1)$$

i.e., with increasing velocity the effective viscosity can other grow or drop (since the velocity can have any sign), which occurs in the given case: up to inflection point of the *S*-shaped curve, the effective viscosity grows (this was found in [2]), after which it starts to decrease.

Then in view of (1) and assuming $r = \sqrt{k/m}$ we write the law of filtration as

$$v = -\frac{k}{\eta_0\left(a + \dfrac{b\tau v\sqrt{m}}{\sqrt{k}} + \dfrac{c\tau^2 v^2 m}{k}\right)}\,\frac{\partial P}{\partial x}\,.$$

Finally, we represent the law of filtration for relaxing liquids of the given class as

TABLE 1. Calculated Values of the Basic Parameters of Filtration of Relaxing Liquids in Porous Media

Filtration of a polymer solution										Filtration of a water-oil emulsion				
homogeneous porous medium					heterogeneous porous medium					heterpgeneous porous medium				
k, μm^2	$\dot{\gamma}_\kappa$ 1/sec	$\dot{\gamma}_0$, 1/sec	Kch_0	v_k, m/sec	k, μm^2	$\dot{\gamma}_\kappa$ 1/sec	$\dot{\gamma}_0$, 1/sec	Kch_0	v_k, m/sec	k, μm^2	$\dot{\gamma}_k$, 1/sec	$\dot{\gamma}_0$,, 1/sec	Kch_0	v_k, m/sec
3.0	100	470	0.33	4.7×10^{-5}	3.0	100	710	0.33	1.17×10^{-4}	3.0	2.7	18	0.55	3.7×10^{-6}

$$-\frac{\partial P}{\partial x} = av + bv^2 + cv^3 . \tag{2}$$

The S-shaped curves that are obtained by us and are described by the single dependence shown in Fig. 3b can be approximated with an error of about 5% by the model:

$$\Delta P_0 = v\,(3.5v - 2.5v^2) ,$$

where $\Delta P_0 = \Delta P k_1$; $k_1 = k/\eta l$; l is the specimen length.

It is pertinent to note that the addition of quadratic and cubic terms to the law of filtration is due to not only a better approximation of the $\Delta P - v$ dependence at the expense of increasing the number of empirical coefficients [12] but also to the physical features of the filtration. The linear term accounts for the governing influence of viscosity forces, the quadratic term reflects that of the nonequilibrium properties of the filtration flow, and the cubic term accounts for the governing influence of mass transfer between the high- and low-permeability portions (i.e., the governing influence of the properties of the porous medium).

Therefore, elastic properties of the relaxing liquids manifest themselves in the heterogeneous porous media, which is expressed in the replacement of the power law of filtration by a S-shaped one in going from a homogeneous porous medium to a heterogeneous medium.

To confirm this reasoning, we estimated the critical shear rate $\dot{\gamma}_{cr}$ beyond which elastic forces prevail over viscous forces and the critical value of Kch (Kch_0), which were determined by us by Malkin's procedure [13] on the basis of data of rotational viscosimetry, as well as of the shear rate $\dot{\gamma}_0$ for which dynamic equilibrium sets in, which is little dependent on the rate of filtration (Newtonian flow with minimum Newtonian viscosity).

The shear rate $\dot{\gamma}_0$ for which Newtonian flow sets in (the shear rate that corresponds to the maximum Newtonian viscosity) was determined by the formula:

$$\dot{\gamma}_0 = \frac{v}{mr} ,$$

where $r = \sqrt{kc'/m}$ is the average radius of a pore channel, i.e.,

$$\dot{\gamma}_0 = \frac{v}{\sqrt{kmc'}} = \frac{Q}{F\sqrt{kmc'}} . \tag{3}$$

Here F is the cross-sectional area of the porous medium; c' is the crookedness of the porous medium (it is equal to 1 for the homogeneous medium and to 2.5 for the heterogeneous media).

Clearly, determination of the average radius of the pore channels and the average shear rate from formula (3) is approximate; however, it enables us to correlate rotational viscosimetry data to the investigations in the porous medium. The results of calculations for some experiments are presented in Table 1, from which it is evident that data of the experiments agree with the calculations. Thus, for the polymer solution (see Fig. 1, curve 1), Newtonian flow sets in when $\dot{\gamma}_0 > 450$ 1/sec, and for the water-oil emulsion (curve 2), when $\dot{\gamma}_0 > 15$ 1/sec, which confirms the above qualitative explanation of the process.

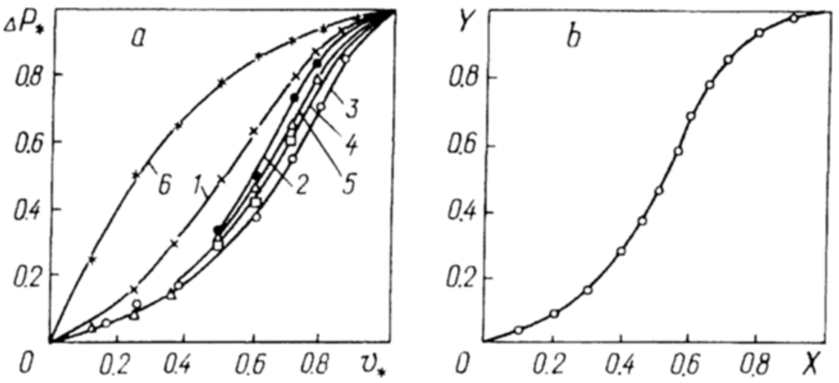

Fig. 4. ΔP_* as a function of v_* (a) and Y as a function of X (b) for a variable ratio of layer permeability K_0: 1) 5.0; 2) 7.5; 3) 10.0; 4) 20.0; 5) 30.0; 6) 1.0.

Noteworthy is the experimental fact obtained by us that a solution of asphaltenes with molecular-order structural elements [14], despite its viscosity as a function of velocity being similar to the dependences shown in Fig. 1, for filtration in the heterogeneous porous medium (the average pore size is equal to $10^{-5}-10^{-6}$ m, i.e., it is considerably larger than the structural elements of the liquid) has a power-law rather than an S-shaped $\Delta P - v$ dependence.

Next, in the second series of the experiments, we determined the influence of the ratio of the layers of a macroheterogeneous bed on the $\Delta P - v$ dependence. The results of the experiments are given in dimensionless coordinates (see Fig. 4a), where $\Delta P_* = \Delta P/\Delta P_\infty$, $v_* = v/v_\infty$. As the figure shows, with an increase in K_0 to 10 the area under the S-shaped curves decreases and then starts to grow again, approaching curve 1, which accounts for the filtration in the homogeneous porous medium. The result obtained becomes clear under the hypothetical assumption of impermeability of one layer. Treatment of the obtained curves in asymptotic coordinates also showed their similarity, i.e., altering the conditions did not lead to a qualitative change in the process (Fig. 4b).

Therefore, the results obtained permit the following conclusions:

1. In filtration of relaxing liquids (whose viscosity decreases with an increasing shear rate according to a power law) in porous media, the character of the $\Delta P - v$ dependence changes from power for a homogeneous porous medium to S-shaped for macro- and microheterogeneous porous media. Elastic properties of the relaxing liquids appear in filtration in the heterogeneous porous media. Allowance for this effect will enable us to improve the efficiency of application of relaxing systems in technological processes.
2. We proposed empirical dependences $\Delta P = f(v)$ for this class of relaxing liquids in homogeneous and heterogeneous porous media that can be used in practical calculations.
3. The ratio of the permeabilities of the high- and low-permeability portions of a heterogeneous porous medium has a substantial effect on the quantitative indexes of the $\Delta P - v$ dependence.

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NOTATION

Q, flow rate of the liquid; η, effective viscosity; η_0, maximum Newtonian viscosity; K, permeability; m, porosity; K_0, permeability ratio of layers; r, average size of pore channel; $\dot{\gamma}_{cr}$, critical shear rate; $\dot{\gamma}_0$, shear rate at which Newtonian flow sets in again.

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